4-6 Mathematics Research

Students' Attitude Toward Mathematics

(NCTM, 2000)

- Nearly three-quarters of U.S. fourth graders report liking mathematics (Silver, Strutchens, and Zawojewski 1997). They find it practical and believe what they are learning is important.
    - If the mathematics studied in grades 3-5 is interesting and understandable, the increasingly sophisticated mathematical ideas at this level can maintain students' engagement and enthusiasm.
        - But if their learning becomes a process of simply mimicking and memorizing, they can soon begin to lose interest.
    - Instruction at this level must be active and intellectually stimulating and must help students make sense of mathematics.

Cognitive Demand of the Fourth, Fifth, and Sixth Grade Curriculum

(NCTM, 2000)

- The mathematical investigations and conversations of students in these grades frequently include elements of algebraic reasoning.
    - These experiences and conversations provide rich contexts for advancing mathematical understanding and are also an important precursor to the more formalized study of algebra in the middle and secondary grades.

(NRC, 2001)

- For both reading and mathematics, children's performance at the end of elementary school is an important predictor of their ultimate educational success.

- If they have not mastered certain basic skills, they can expect problems throughout their schooling and later.

- Much of the difficulty that students experience when they first encounter algebra is symptomatic of the cognitive challenges inherent in moving from one mode of thinking to another, from arithmetic reasoning to algebraic reasoning.

- By focusing on ways to use the elementary and middle school curriculum to support the development of algebraic reasoning, these efforts attempt to avoid the difficulties many students now experience and to lay a better foundation for secondary school mathematics (Blanton and Kaput, 2000; Carpenter and Levi, 1999; Carraher, Brizuela, and Schliemann, 2000; Schliemann, Carraher, and Brizuela, 2000).

Teacher Content Knowledge

(NCTM, 2000)

- Because of the increasing mathematical sophistication of the curriculum in grades 3-5, the development of teachers' expertise is particularly important.
    - Teachers need to understand both the mathematical content for teaching and students' mathematical thinking.
    - Many elementary teacher preparation programs require minimal attention to mathematics content knowledge.
        - For successful implementation of these Standards, it is essential that the mathematical expertise of teachers be developed, whatever model is used.

(NRC, 2001)

- In the last 15 years, researchers have investigated how teachers' mathematical knowledge shapes the way they teach. In general, the researchers found that teachers with a relatively weak conceptual knowledge of mathematics tended to demonstrate a procedure and then give students opportunities to practice it.
    - Not surprisingly, these teachers gave the students little assistance in developing an understanding of what they were doing (Ball, 1991; Leinhardt and Smith, 1985).
    - When the teachers did try to provide a clear explanation and justification, they were not able to do so. In some cases, their inadequate conceptual knowledge resulted in their presenting incorrect procedures (Leinhardt and Smith, 1985; Putnam, Heaton, Prawat, and Remillard, 1992).

- Proficiency in mathematics teaching has parallels to proficiency in mathematics.

- Teachers are unlikely to be able to provide an adequate explanation of concepts they do not understand, and they can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in a single way.

Professional Development in Mathematics

(NRC, 2001)

- Teachers' professional development should be high quality, sustained, and systemically designed and deployed to help all students develop mathematical proficiency. Schools should support, as a central part of teachers' work, engagement in sustained efforts to improve their mathematics instruction. This support requires the provision of time and resources.

- Effective programs of teacher preparation and professional development help teachers understand the mathematics they teach, how their students learn that mathematics, and how to facilitate that learning.

- Just as students' opportunities to learn mathematics effectively have been made insufficient, so have teachers' opportunity to learn more about mathematics, students' learning and thinking, and their teaching practice.

- They (teachers) need access to resources and expertise that will assist them in improving their instruction, including access to mathematics specialist in every elementary school.

- Improving students' learning depends on the capabilities of classroom teachers. Although children bring important mathematical knowledge with them to class, most of the mathematics they know is learned in school and depends on those who teach it to them.

- Teachers cannot automatically know how to teach more effectively. Learning to teach well cannot be accomplished once and for all in a preservice program; it is a career-long challenge.

- If the United States is serious about improving their instruction, including students' mathematics learning, it has no choice but to invest in more effective and sustained opportunities for teachers to learn.

- The coordination of curriculum, instructional materials, assessment, instruction, professional development, and school organization around the development of mathematical proficiency should drive school improvement efforts.

- In the 1996 NAEP mathematics assessment, teachers were asked how many hours of professional development they had received in the previous 12 months. Nationally, 28% of the fourth graders in the sample had teachers who had received 16 or more hours of professional development in mathematics; for eight graders, the percentage was 48.

- Professional development in mathematics needs to be sustained over time that is measured in years, not weeks or months, and it needs to involve a substantial amount of time each year. Our recommendations to raise the level of professional development are as follows:
    - Local education authorities should give teachers support, including stipends and released time, for sustained professional development.
    - Providers of professional development should know mathematics and should know about students' mathematical thinking, how mathematics is taught, and teachers' thinking about mathematics and their own practice.

- Organizations and agencies that fund professional development in mathematics should focus resources on multi-year, coherent programs. Resources of agencies at every level should be marshaled to support substantial and sustained professional development.

Mathematics Instruction vs. Reading Instruction

(NRC, 2001)

- For both reading and mathematics, children's performance at the end of elementary school is an important predictor of their ultimate educational success.

- To ensure that students having reading difficulties get prompt and effective assistance outside the regular school program, the reading community has developed a variety of intervention program designed to address the problems students are having and to bring them back rapidly into the regular program.

- Although there is much "remediation" done as part of school mathematics instruction in grades K to 8 and beyond, there are not nearly so many supplementary interventions in mathematics as there are in reading.
- There is very little in the way of "mathematics recovery" that provides early targeted enrichment in mathematics to help students overcome special difficulties.

- One difference between reading and mathematics is that, after a certain point, reading requires little explicit instruction: Once children have acquired basic principles and skills for reading, they use those skills in the service of other activities, to learn about history, literature, or mathematics, for example. Their skills can always be polished and instruction given on interpreting a text, but they need no further explanations and demonstrations of reading by others.
    - Furthermore, they practice and develop their reading throughout their lives, both inside and outside of school. As is the case for reading, students develop some basic concepts and practices in mathematics outside of school, but a new and unfamiliar topic in mathematics—say, the division of fractions—usually cannot be fully grasped without some assistance from a text or a teacher.

- Reading uses a core set of representations.
    - In U.S. Schools, the English alphabet writing system, once learned, enables the student to read and decode any English sentence, although of course not necessarily to understand its meaning.
    - Graphs, pictures, and signs to also need to be read, but the core symbols are the alphabet.

- Mathematics, in contrast, has many types and levels of representation. In fact, mathematics can be said to be *about* levels of representation, which build on one another as the mathematical ideas become more abstract. For example, the increasing focus on algebra during the school years builds facility with more abstract levels of representation.

- Another characteristic of learning to read is the vast variation among children in their exposure to literature outside of school, as well as in the amount of time they spend reading. Studies on the development of reading (Wagner and Stanovich, 1996) have shown that variations in children's reading skill are associated with large differences in reading experience. Children at the 80^{th} percentile in reading level were estimated to average more than 20 times as much reading per day as children at the 20^{th} percentile (Anderson, Wilson, and Fielding, 1988).
    - Similar data are not available for mathematics, but differences in the amount of time spent doing mathematics are likely to be less than for reading.
        - This suggests that direct school-based instruction may play a larger part in most children's mathematical experience than it does in their reading experience.
        - If so, the consequences of good or poor mathematics instruction may have an even greater effect on children's proficiency than is the case with reading.

- An important recent change in American education is the increased emphasis on ensuring that all children achieve a basic level of competence in reading during the course of elementary school.

    - Success in school also depends on establishing good mathematical competence in the early elementary grades, yet mathematics instruction has not received the same sustained emphasis.
    - Schools generally lack a mathematics specialist corresponding to the reading specialists who provide instruction and assist children having difficulties with the subject.

- Many school districts have revised their schedules and their curriculum programs to ensure that adequate reading instruction is given in the elementary grades; mathematics instruction has yet to receive similar attention.

References

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*These two publications have synthesized the mathematics research in many journal articles and have cited them within the publications.